

Non-perturbative microscopic theory of superconducting fluctuations near a quantum critical point

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We consider an inhomogeneous anisotropic gap superconductor in the vicinity of the quantum critical point, where the transition temperature is suppressed to zero by disorder. Starting with the BCS Hamiltonian, we derive the Ginzburg-Landau action for the superconducting order parameter. It is shown that the critical theory corresponds to the marginal case in two dimensions and is formally equivalent to the theory of an antiferromagnetic quantum critical point, which is a quantum critical theory with the dynamic critical exponent, $z = 2$. This allows us to use a parquet method to calculate the non-perturbative effect of quantum superconducting fluctuations on thermodynamic properties. We derive a general expression for the fluctuation magnetic susceptibility, which exhibits a crossover from the logarithmic dependence, $\chi(T, H, n) \propto \ln[\delta n(T, H)]$, valid beyond the Ginzburg region to $\chi(T, H, n) \propto \ln^{1/5}[\delta n(T, H)]$ valid in the immediate vicinity of the transition (where δn is the deviation from the critical disorder concentration). We suggest that the obtained non-perturbative results describe the low-temperature critical behavior of a variety of diverse superconducting systems, which include overdoped high-temperature cuprates, disordered p -wave superconductors, and conventional superconducting films with magnetic impurities.

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The problem of the critical behavior of itinerant electronic systems in the vicinity of quantum phase transitions has been the subject of intense theoretical investigations in recent years [1, 2, 3, 4, 5]. Most theoretical studies of the problem are based on effective field theories, which describe critical fluctuations near the transition. The bare Ginzburg-Landau coefficients of these models are usually treated as phenomenological parameters, which often remain undefined due to the lack of a reliable microscopic theory of the transition. Indeed, while Fermi liquid theory predicts and classifies possible intrinsic instabilities, there is usually no controlled approach to access the transition point on the basis of a fermionic Hamiltonian. The notable exception is a superconducting instability, which is well-described by the perturbative Bardeen-Cooper-Schrieffer (BCS) theory. A strong magnetic field or disorder effects may suppress the superconducting transition temperature to zero and therefore lead to a superconducting quantum critical point (QCP), which can be studied on the basis of the BCS model.

The critical behavior of a superconducting system near the transition is governed by superconducting fluctuation effects [6], which physically are due to uncondensed fluctuating Cooper pairs, which co-exist with electronic excitations. A perturbative theory of classical superconducting fluctuations was developed by Aslamazov and Larkin in 1968 [7]. More recently, Larkin and the author [8] considered quantum superconducting fluctuations near the magnetic-field-tuned QCP. In both cases, it was found that Gaussian fluctuations strongly affect thermodynamics and transport near the critical point. Even though the Aslamazov-Larkin theory has been very successful in describing a variety of experiments, it is strictly speaking not a critical theory, but a Gaussian perturbation theory, which assumes that the fluctuating Cooper pairs do not interact, and applies only far enough from the transition (beyond the Ginzburg region). To the best of the author's knowledge, there is no physically relevant example of a non-perturbative

treatment of superconducting fluctuations.

In this paper, we develop such a non-perturbative microscopic theory for a disordered anisotropic gap superconductor near the disorder-tuned QCP. We derive the corresponding critical theory, which is shown to be identical to the Hertz-Moriya-Millis theory [3, 4] of an antiferromagnetic QCP in two dimensions. We find that the dimensionless bare quartic coupling (which characterizes the interaction between superconducting fluctuations) is a small parameter of the order of the inverse conductance and this quartic term becomes marginally irrelevant at the transition. This allows us to perform non-perturbative parquet resummation of the leading logarithms and find the exact critical behavior of the magnetic susceptibility near the transition. The latter crosses over from $\chi(T, n) \propto \ln[n - n_c(T)/n]$ (which is the quantum analogue of the Aslamazov-Larkin result valid far enough from the transition) to the critical behavior $\chi(T, n) \propto \ln^{1/5}[n - n_c(T)/n]$, which holds within the quantum Ginzburg region.

Let us consider a disordered electron system with a weak attraction in the l -wave channel, described by the Hamiltonian

$$\hat{H} = \sum_{\mathbf{p}} \hat{\psi}_{\mathbf{p}}^\dagger \xi_{\mathbf{p}} \hat{\psi}_{\mathbf{p}} + \frac{1}{2} \sum_{\mathbf{p}, \mathbf{p}', \mathbf{q}} V(\mathbf{p}, \mathbf{p}') \hat{\psi}_{\mathbf{p}}^\dagger \hat{\psi}_{-\mathbf{p}+\mathbf{q}}^\dagger \hat{\psi}_{\mathbf{p}'} \hat{\psi}_{-\mathbf{p}'+\mathbf{q}} + \hat{H}_{\text{dis}}, \quad (1)$$

where $\xi_{\mathbf{p}} = E(\mathbf{p}) - \mu$ is the spectrum, \hat{H}_{dis} represents a disorder potential (which we assume to be due to Poisson distributed short-range impurities), the interaction is $V(\mathbf{p}, \mathbf{p}') = -\lambda_l u_l(\phi) u_l(\phi)$, λ_l is the interaction constant, and $u_l(\phi)$ defines the symmetry of the gap. In what follows we assume an unusual pairing symmetry (e.g., d -wave), so that $\overline{u_l} = \int_0^{2\pi} u_l(\phi) d\phi / (2\pi) = 0$ and $\overline{u_l^2} = 1$. We suppress spin indices throughout the paper. Next, we introduce an anisotropic order parameter and allow for its spatial and temporal fluctuations

$$\Delta_{\mathbf{k}}(\mathbf{r}, \tau) = T \sum_{\mathbf{k}', \mathbf{q}, \omega_n} V(\mathbf{k}, \mathbf{k}') F\left(\mathbf{k}' - \frac{\mathbf{q}}{2}, \mathbf{k} + \frac{\mathbf{q}}{2}; \omega_n\right) e^{i\mathbf{q} \cdot \mathbf{r} - i\omega_n \tau}, \quad (2)$$

where τ is the Matsubara time and $F(\mathbf{r}, \tau; \mathbf{r}', \tau') = -\langle T_\tau \hat{\psi}(\mathbf{r}, \tau) \hat{\psi}(\mathbf{r}', \tau') \rangle$ is the Gor'kov's Green's function. Following Ref. [9], we assume that the symmetry of the gap is preserved, $\Delta_{\mathbf{k}}(\mathbf{r}, \tau) = \Delta(\mathbf{r}, \tau) u_l(\phi_{\mathbf{k}})$, and integrate out the one-particle degrees of freedom to obtain the following effective action for the order parameter near the transition

$$S[\Delta] = \nu \int_Q \left[\frac{\tau_c(T, H) - \tau}{\tau_{c0}} + |\omega| \tau + D \mathbf{q}^2 \tau \right] |\Delta(Q)|^2 \quad (3) \\ + \frac{B}{2} \int_{Q_1, Q_2, Q_3} \Delta^*(Q_1) \Delta(Q_2) \Delta^*(Q_3) \Delta(Q_1 + Q_3 - Q_2),$$

where ν is the density of states at the Fermi line and we use a three-vector $Q = (\omega_n, \mathbf{q})$ and the notation $\int_Q f(Q) \equiv T \sum_{\omega_n} \int d^2 \mathbf{q} / (2\pi)^2 f(\omega_n, \mathbf{q})$ for brevity (ω_n is the bosonic Matsubara frequency). The general expression for the quartic coefficient B (at an arbitrary temperature and disorder strength) was derived by the author in Ref. [9]. Near the QCP, $T\tau_{c0} \rightarrow 0$, it reduces to $B = \nu \tau^2 (\overline{u_l^4} - 1/3)$ [c.f., $B = 7\overline{u_l^4} \zeta(3) \nu / (8\pi^2 T^2)$ in the classical limit, $T\tau_{c0} \gg 1$], where the overline implies an averaging over the directions on the Fermi line. *E.g.*, in a d -wave superconductor, we get $\overline{u_d^4} = 3/2$. In Eq. (3), the critical scattering time (or disorder concentration, $n_c \sim \tau_c^{-1}$) determines the superconducting transition point and is a function of temperature and magnetic field. To find the dependence on the latter, we can just replace the Cooper-pair momentum \mathbf{q} with the operator $\hat{\pi} = [-i\nabla - \frac{2e}{c} \mathbf{A}(\mathbf{r})]$ and evaluate the matrix element corresponding to the lowest Landau level in Eq. (3) by replacing $D\mathbf{q}^2$ with $\langle 0 | D\hat{\pi}^2 | 0 \rangle = 2eDH/c$. The full three-dimensional temperature-disorder-magnetic field phase diagram is determined by the equation

$$\ln \frac{T_{c0}}{T} = \psi \left\{ \frac{1}{2} + \frac{1}{4\pi T} \left[\frac{1}{\tau_c(T, H)} + 2 \frac{e}{c} DH \right] \right\} - \psi \left(\frac{1}{2} \right), \quad (4)$$

where $\psi(z)$ is the logarithmic derivative of the gamma-function. The zero-temperature solution of this equation represents a line of quantum critical points. In what follows, we consider the low-temperature limit and small magnetic fields. In this case, the asymptotic behavior of the critical scattering time is

$$\frac{\tau_c(T, H) - \tau_{c0}}{\tau_{c0}} = 2\pi g \omega_c \tau_{c0} + \frac{2\gamma^2}{3} \left(\frac{T}{T_{c0}} \right)^2, \quad (5)$$

where $g = E_F \tau / \pi \gg 1$ is the dimensionless conductance, $\omega_c = eH/(mc)$ is the cyclotron frequency, $1/\tau_{c0} = \pi T_{c0}/\gamma$ is the critical scattering rate corresponding to $T = 0$ and $H = 0$, $\gamma \approx 1.781$ is the Euler's constant, and $T_{c0} \propto \exp[-1/(\lambda_l \nu)]$ is the BCS transition temperature in a clean system.

Now, we consider the ultra-low-temperature regime in which quantum rather than thermal fluctuations determine the critical behavior of the system, which corresponds to the limit $r(T, H, \tau) = [\tau_c(T, H) - \tau] / \tau \gg 2\pi T \tau$. In this case, we can replace the Matsubara sums in Eq. (3) with the integrals over the frequency. It is also convenient to introduce new variables

$r = [\tau - \tau_c(T, H)] / \tau$ and $\rho_0 = B/(8D\tau^2\nu^2)$ and rescale the parameters in Eq. (3) as follows: $k_0 = \omega \tau$, $\mathbf{k} = \sqrt{D\tau} \mathbf{q}$, and $\phi = \sqrt{2\nu/(D\tau^2)} \Delta$. Using these dimensionless variables, we arrive to the following quantum critical action

$$S[\phi] = \frac{1}{2!} \int_k [r + |k_0| + \mathbf{k}^2] |\phi(k)|^2 \quad (6) \\ + \frac{\rho_0}{4!} \int_{k_1, k_2, k_3} \phi^*(k_1) \phi(k_2) \phi^*(k_3) \phi(k_1 + k_3 - k_2),$$

where the parameter $\rho_0 = (3\overline{u_l^4} - 1)/g \ll 1$ is essentially the inverse conductance and thus is small. We emphasize that the above action is not a phenomenological effective theory, but a microscopic result, which follows from the basic BCS Hamiltonian (1). The latter contains just one phenomenological parameter - the Cooper channel interaction constant, which is encoded in a non-universal "clean" transition temperature, T_{c0} . The exact mechanism of unconventional superconductivity and the exact value of T_{c0} are not important for the problem at hand, which deals with disorder-induced suppression of superconductivity that happens at the diffusive rather than ballistic length-scales. Another important observation is that action (6) is identical to the Hertz-Moriya-Millis theory [3, 4] of the two-dimensional antiferromagnetic QCP. The latter is known to be a marginal theory [10]. Since the dimensionless coupling constant in Eq. (6) is small, the renormalization group (RG) treatment of the theory is an asymptotically exact approach.

The upper cut-off in the RG approach is determined by the applicability of the diffusion approximation, which is reasonable as long as $|\omega_{\max}| \sim Dq_{\max}^2 \lesssim 1/\tau$. In terms of the dimensionless three-momentum $k = (k_0, \mathbf{k})$, this condition implies $|k_{\max}| = \Lambda \sim 1$. We use this value of the cut-off and the usual RG scheme [4], to obtain the scattering amplitude of fluctuations $\rho(k_1, k_2, k_3, k_4)$. In principle, the latter depends on all external momenta, but within the logarithmic accuracy, we can just consider it to be a function of a single variable $\rho(k = \max\{k_i\})$, for which we get the following equation [4, 10] $\rho(k) = \rho_0 - \frac{5}{3} \int_k \rho^2(k') (r + |k'_0| + \mathbf{k}'^2)^{-2}$, which leads to the "zero-charge" behavior of $\rho(k)$

$$\rho(k) = \frac{\rho_0}{1 + \frac{5\rho_0}{12\pi^2} |\ln[r + |k_0| + \mathbf{k}^2]|}. \quad (7)$$

Since the bare scattering amplitude is determined by the inverse conductance, it is alluring to interpret this "zero charge" result as a flow of the resistivity to zero in the superconducting phase. In fact, a naïve calculation of the Aslamazov-Larkin conductivity diagram [7] in which the current vertices are taken to be independent of frequencies does lead to the logarithmic behavior of the corresponding correction to the conductivity [11], which would be consistent with the above-mentioned interpretation. However, a more careful analysis of the Aslamazov-Larkin diagram shows that this standard approximation [6] (valid near a classical transition) breaks down near a QCP (see also Ref. [8], where this complication was

first pointed out) and a full frequency and momentum dependence of the current vertices is needed to get a correct result. To calculate transport properties turns out to be very difficult due to the problem of analytical continuation of the propagators and vertices (which here are very complicated non-analytic functions of two complex variables). However, to obtain thermodynamic properties is rather straightforward and can be done non-perturbatively on the basis of the action (6).

Below, we consider the fluctuation magnetic susceptibility near the quantum phase transition.

$$\chi = -\frac{1}{V} \frac{\partial^2 F}{\partial H^2} = -\frac{1}{V} \left(\frac{\partial r}{\partial H} \right)^2 \frac{\partial^2 F}{\partial r^2}, \quad (8)$$

where the $r(H)$ -dependence is given by Eq. (5), which yields $(\partial r / \partial H) = e\tau^2(v_F^2/c^2)$. Now, we follow Larkin and Khmel'nitskii [12] and notice that the second derivative of the free energy in (8) is the exact polarization operator given by

$$\Pi(r) = \frac{1}{D\tau^2} \int \frac{d^3k}{(2\pi)^3} \frac{\mathcal{T}^2(k)}{(r + |k_0| + \mathbf{k}^2)^2}, \quad (9)$$

where $\mathcal{T}(k)$ is the vertex function, which is determined by

$$\mathcal{T}(k) = 1 - \frac{2}{3} \int_{\max\{|k_0|, |\mathbf{k}|\}}^1 \frac{d^3k'}{(2\pi)^3} \frac{\mathcal{T}(k')\rho(k')}{(r + |k'_0| + \mathbf{k}'^2)^2}. \quad (10)$$

Eqs. (8), (9), and (10) lead to the magnetic susceptibility per unit volume

$$\chi = -\frac{12\pi g^2}{(3u_l^4 - 1)} \frac{e^2}{d\hbar mc^2} \left\{ \left[1 + \frac{5(3\overline{u}_l^4 - 1)}{12\pi^2 g} \ln \frac{1}{r(T, H, \tau)} \right]^{1/5} - 1 \right\}, \quad (11)$$

where d is the thickness of the film (or interlayer distance in the case of a layered superconductor) and the proximity to the transition $r(T, H, \tau)$ is given by Eq. (5). Eq. (11) has two asymptotic regimes,

$$\chi \approx -\frac{e^2}{d\hbar mc^2} \begin{cases} (g/\pi) |\ln r|, & \text{if } 1 \ll |\ln r| \ll g; \\ 7.358 g^{9/5} |\ln r|^{1/5}, & \text{if } |\ln r| \gg g, \end{cases} \quad (12)$$

where the second numerical coefficient corresponds to the d -wave case. The former (finite r) behavior is the regime of regular Gaussian perturbation theory. The later regime of $r \rightarrow 0$ is clearly non-perturbative and becomes asymptotically exact in the very vicinity of the QCP. Note that the fluctuation diamagnetism exceeds the Landau susceptibility $\chi_{\text{Landau}} = -e^2/(12\pi d\hbar mc^2)$ by orders of magnitude even far from the transition. We reiterate that there is no contradiction here [6] since Eq. (11) should be viewed as a correction to the perfect diamagnetic response of a superconductor (not to the Pauli/Landau terms in a normal metal). We also note here that the effect of quantum fluctuations on other thermodynamic properties, such as the specific heat, are unremarkable since the critical density of disorder depends on the temperature as $r(T) \propto T^2$.

Now we discuss the crossover between the quantum fluctuation and classical fluctuation regimes. Clearly at large temperatures, the non-perturbative RG treatment breaks down, because the integral over frequency (which makes the quantum problem effectively 4-dimensional) is replaced with the Matsubara sum and only the $\omega_n = 0$ term plays a role near the transition; thus, we recover the two-dimensional model. The crossover between the two behaviors (here we are talking about the linear-response $H \rightarrow 0$ magnetic susceptibility) happens around $r \sim 2\pi T\tau = 2\gamma(T/T_{c0})$. The general expression for the leading order correction to the fluctuation susceptibility is determined by $\chi^{(1)} = -(\partial r / \partial H)^2 T \sum_{\omega_n} \int d^2\mathbf{q} / (2\pi)^2 [r + |\omega_n|\tau + D\mathbf{q}^2\tau]^{-2}$, which leads to the result

$$\chi^{(1)}(T) = -\frac{e^2}{d\hbar mc^2} \frac{g}{\pi} \left\{ \psi \left[\frac{1}{2\pi T\tau} \right] - \psi \left[\frac{r(T)}{2\pi T\tau} \right] - \frac{\pi T\tau}{r(T)} \right\}, \quad (13)$$

The low-temperature quantum limit in Eq. (13) reproduces the logarithmic asymptotic of Eq. (12), while the ‘‘high-temperature’’ limit $r \ll (T/T_{c0}) \ll 1$ leads to the classical Aslamazov-Larkin-type power law divergence

$$\chi^{(1)} = -\frac{3g}{4\pi\gamma} \frac{e^2}{d\hbar mc^2} \frac{T_{c0}}{T - T_c(\tau)}, \quad (14)$$

where we introduced a scattering time-dependent transition temperature [which is the inverse function of $\tau_c(T)$ used earlier, see Eq. (5)]. We emphasize that Eq. (14) corresponds to the limit $(T/T_{c0}) \ll 1$. However, at higher temperatures and in particular near T_{c0} (clean limit), the familiar Aslamazov-Larkin power law and all parameters are preserved and only the numerical coefficient changes.

Fig. 1 summarizes the behavior of fluctuations in the vicinity of the disorder-induced superconductor-metal transition in an anisotropic gap superconductor. The solid black line represents the phase boundary between the superconducting and metallic phases. The hatched area is the Ginzburg region, where the fluctuations interact strongly and Aslamazov-Larkin theory breaks down. Interestingly, the width of the quantum Ginzburg region, $\text{Gi}_Q = \exp(-g)$, is much smaller than that of the classical Ginzburg region, $\text{Gi}_C = 1/g$. The two dashed lines in Fig. 1 separate the classical and quantum fluctuation regimes. The classical regime is effectively a two-dimensional theory ($d_{\text{eff}}^C = 2$) in which the leading order perturbative correction diverges as a power law, $\chi_C^{(1)} \propto -(T - T_c)^{-1}$. The effective dimensionality in the quantum regime is $d_{\text{eff}}^Q = 4$ and the leading order correction to susceptibility (and likely conductivity [11]) is logarithmic, $\chi_Q^{(1)} \propto \ln(n - n_c)$. The region in between the two dashed lines represents a crossover between the quantum and classical behavior [3, 4, 13] and in some sense describes a crossover between the effective dimensionality $d_{\text{eff}}^Q = 4$ in the former to the effective dimensionality $d_{\text{eff}}^C = 2$ in the latter. The leading order correction to thermodynamics in this regime is described by the non-linear function in Eq. (13), which we found to be a smooth function with no remarkable properties. The exact

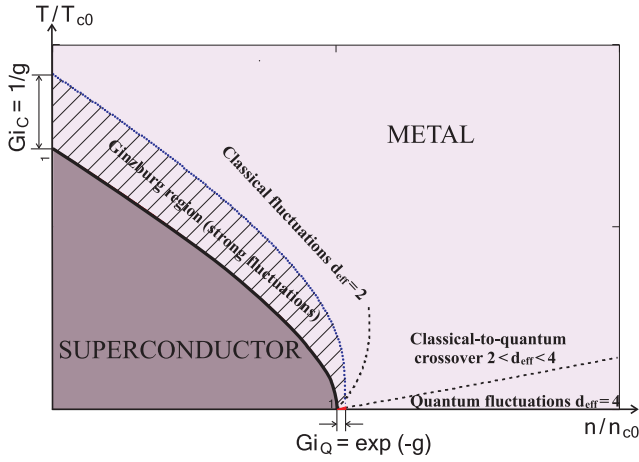


FIG. 1: (Color online) This figure illustrates the phase diagram (transition temperature vs. concentration of disorder) and various fluctuation regimes. The hatched region is the regime of strong superconducting fluctuations. The dashed lines determine the quantum, classical, and quantum-to-classical crossover regions in the fluctuation thermodynamics. See text for a more detailed discussion.

non-perturbative behavior in the classical and crossover regions is unknown, but some insight can be obtained via an ϵ -expansion [4, 10]. In the quantum Ginzburg region, the RG approach is asymptotically exact and leads to the susceptibility, which behaves as $\chi_Q \propto \ln^{1/5}(n - n_c)$.

We note that in a disordered system, the latter non-perturbative behavior may be smeared out by the Griffiths effects due to mesoscopic fluctuations of disorder. There always exist superconducting puddles, which appear in the local areas, where the concentration of impurities is smaller than the average [9, 14, 15]. At low temperatures, these puddles are connected via Josephson tunneling, which is long-range in real space $J(r_{ij}) \propto |\mathbf{r}_i - \mathbf{r}_j|^{-2}$. A global transition in this random network is governed by phase fluctuations [14]. The corresponding theory is however not a pure random XY-model. First, in a d -wave superconductor (or any other superconductor where the gap vanishes), there always exist gapless quasiparticles with a small density of states, which lead to an unusual intrinsic dissipation and non-local in time Josephson coupling even in the absence of a normal component (the long-time asymptote of both the dissipation and Josephson kernels behave as [16] $\alpha(\tau) \propto J(\tau) \propto |\tau|^{-3}$). For superconducting puddles surrounded by a normal metal with a large electronic density of states, the dissipation kernel acquires the usual Caldeira-Leggett form $\alpha(\tau) \propto |\tau|^{-2}$, while the Josephson term remains long-range in time $J(\tau) \propto |\tau|^{-3}$. Within a mean field approach, one can estimate [11] that the QCP (5) is shifted by $\delta r \sim g^{-1}$ and therefore the quantum Ginzburg region gets absorbed by the Griffiths phase. A complete theory of the quantum Griffiths phase will be presented elsewhere. We note here that while it is not clear whether the evasive Griffiths phase can be observed in real experiment, the quantum fluctuation effects described by the general equation (11) should definitely be accessible at least for $\delta r \gtrsim g^{-1}$ and possi-

bly in an even wider range of parameters.

The physical systems where these fluctuations may be experimentally observed include disordered superconducting films with an unusual pairing symmetry and possibly high- T_c cuprates in the vicinity of the termination of the superconducting dome in the overdoped regime. Independently of the nature of the transition in the overdoped high- T_c materials, the effective theory still should have the form (6) [but the physical meaning of the transition parameter r in (6) may be different] and therefore all functional dependencies should remain the same. We also note here that recent STM experiments, (see e.g. Ref. 17) suggest that disorder may play an important role in high- T_c materials and, if so, should contribute to T_c -suppression [9]. Another superconducting system where the field theory of the transition is identical to (6) is an s -wave superconducting film in which the transition temperature is suppressed to zero by magnetic disorder. Finally, we emphasize that since a superconductor is a perfect diamagnetic material, fluctuation diamagnetism is physically a correction to this perfect diamagnetic behavior and in the vicinity of a superconducting transition, the magnetic susceptibility is always much larger than the Fermi liquid terms. Thus, the experimental verification of the predicted quantum critical behavior of the magnetic susceptibility is certainly feasible.

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